ANALYSIS AND DESIGN OF CLOSED IRREVERSIBLE CYCLES THROUGH FINITE PHYSICAL DIMENSIONS THERMODYNAMICS

by

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Abstract. This paper develops simplifying entropic models of irreversible closed cycles. The entropic models involve the irreversible connections between external and internal main operational parameters with finite physical dimensions. The external parameters are the mean temperatures of external heat reservoirs, the heat transfers thermal conductance, and the heat transfer mean log temperatures differences. The internal involved parameters are the reference entropy of the cycle and the internal irreversibility number. The internal irreversibility number allows the evaluation of the reversible heat output function of the reversible heat input. Thus the cycle entropy balance equation to design the irreversible closed cycles only through external operational parameters might be involved.

Introduction

Development of proof thermodynamic design models has to be performed by several logical stages, synthetically presented below.

1. Defining the reference complete reversible models. This stage is considering Carnot Cycle.
2. Defining the reference endoreversible models. This stage might be well achieved through Finite Physical Dimensions Thermodynamic (FPDT) mathematical models allowing the generalization of design results, not depending on the working fluid nature. For any endoreversible cycle the limitations of the endoreversible Carnot cycle are exceeded through the mean thermodynamic temperature concept.
3. Defining the reference models assessing the irreversibility influence. The equilibrium thermodynamics was completed through mean thermodynamic temperature, exergy and irreversible entropy generation concepts. The FPDT assessments might be completed defining a single concept evaluating priori the overall internal irreversibility and involving the cyclic entropy balance.
4. Defining the optimization methods of reference reversible and irreversible cycles. The optimization methods consider either pure thermodynamic criteria, or CAPEX criteria, or operational costs criteria, or environmental criteria. The more elaborated methods combine different criteria.
5. Defining the reference models for possible interconnected grids of irreversible cycles and the evaluation of performances, energy interactions, investments, operational costs, environmental effects, preservation of natural resources.

2. Mathematical algorithm

The Irreversible Closed Cycles—The Irreversible Energy Efficiency—The Reference Entropy—The Number of Internal Irreversibility

Let us suppose the general basic irreversible thermal systems interacting with the 'environment' by heat transfers, mass transfers and power transfers, see Figure 1.

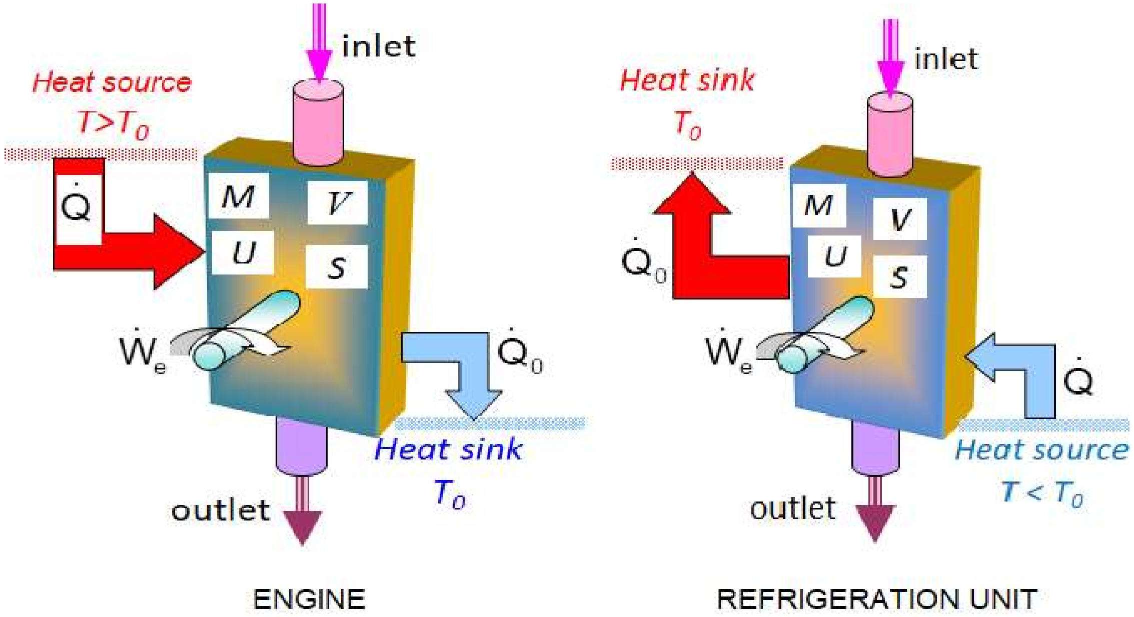

Figure 1. Basic irreversible thermal systems.

M: mass of the working fluid surrounded by the inner walls at a certain operational time.
V: working fluid volume defined by the inner walls at a certain operational time.
U: internal energy of the working fluid surrounded by the inner walls at a certain operational time.
S: entropy of the working fluid surrounded by the inner walls at a certain operational time.

These thermal systems can be analyzed taking into account either the whole irreversibility, internal and external, or only the internal irreversibility. Three types of thermal systems would be analyzed:

I. the enlarged open thermal system comprising three interconnected parts and completely isolated from the universe, i.e., the proper open thermal system deformable under the external pressure which is joined with the external heat transfer reservoirs having known mean temperatures and heat capacities and joined with the deformation work and mass transfer reservoirs having known parameters, pressure, temperature, mass composition and specific energies (enthalpies and entropies including both the chemical and physical parts , kinetic, and potential energies);

II. the enlarged non-deformable closed thermal system has two coupled parts isolated from the universe, the proper closed thermal system joined only with the external heat transfer reservoirs having known mean temperatures and specific heat capacities, and

III. the closed thermal system/cycle considered alone but connected to external heat reservoirs with unknown parameters.

2.1. *Assumptions for the First Case, the Enlarged Thermal Systems*

- Non steady-state enlarged basic open thermodynamic systems, including both the thermal system, and the external heat reservoirs controlling the heat transfers, and the environment allowing the mass transfers and the deformation work transfer under the external pressure, see Figure 1;
- The working fluid is a mixture of different chemical species, the inlet and outlet compositions might be different because of chemical reactions that can appear during the flow through the thermal system, e.g., combustion;
- The inner boundary of the flow path through the thermal system is deformable under the environmental pressure;

Correlating the first law Equation (1) with the second law Equation (2), they can obtain the most general equation of the irreversible power (3) connected to the complete reversible cycle

$$\frac{\partial U}{\partial t} = \left(\dot{Q} - \left|\dot{Q}_0\right|\right) - \dot{W}_e - p_e \frac{\partial V}{\partial t} + \sum_{inlet} \dot{m}\left(h + \frac{c^2}{2} + gZ\right) - \sum_{outlet} \dot{m}\left(h + \frac{c^2}{2} + gZ\right) \quad (1)$$

$$\dot{S}_{gen}^{irrev} = \frac{\partial S}{\partial t} - \left(\frac{\dot{Q}}{T} - \frac{\left|\dot{Q}_0\right|}{T_0}\right) - \sum_{inlet} \dot{m}s + \sum_{outlet} \dot{m}s \geq 0 \quad (2)$$

$$\dot{W}_e = \dot{W}_e^{rev} + \dot{W}_{lost}^{irrev} = \dot{Q}\left(1 - \frac{T_0}{T}\right) + \sum_{inlet} \dot{m}\left((h - T_0 s) + \frac{c^2}{2} + gZ\right) - \sum_{outlet} \dot{m}\left((h - T_0 s) + \frac{c^2}{2} + gZ\right) - \frac{\partial}{\partial t}(U + p_e V - T_0 S) - T_0 \dot{S}_{gen}^{irrev} \quad (3)$$

$$\dot{W}_e^{rev} = \dot{W}_{e,Q}^{rev} + \dot{W}_{e,flow}^{rev} + \dot{W}_{e,storage}^{rev} \quad (4)$$

$$\dot{W}_{e,Q}^{rev} = \dot{Q}\left(1 - \frac{T_0}{T}\right) \quad (5)$$

$$\dot{W}_{e,flow}^{rev} = \sum_{inlet} \dot{m}(h^* - T_0 s) - \sum_{outlet} \dot{m}(h^* - T_0 s) \quad (6)$$

$$\dot{W}_{e,storage}^{rev} = -\frac{\partial}{\partial t}(U + p_e V - T_0 S) \quad (7)$$

$$\dot{W}_{lost}^{irrev} = -T_0 \dot{S}_{gen}^{irrev} \quad (8)$$

where:

- $\dot{Q}, \dot{Q}_0, \dot{W}_e, \dot{W}_e^{rev}, \dot{W}_{lost}^{irrev}$ are the heat transfer rates from the heat source and to the heat sink, the real irreversible power, the complete reversible power and the lost power through irreversibility;
- $p_e \frac{\partial V}{\partial t}$ is the deformation work transfer under the external pressure, p_e is the external pressure and V is the deformable volume of the thermal system;
- $\dot{m}$, h, s are the mass flow rates, the specific enthalpy and the specific entropy inclosing both the chemical and physical parts, compulsory to obey to the first law of thermodynamics and considering all possible internal chemical processes, e.g., combustion;
- $\frac{c^2}{2}, gZ$ are the specific kinetic and potential energies;
- T, T_0 are the mean temperatures of the heat source and of the heat sink;
- $\dot{S}_{gen}^{irrev}$ is the entropy rate generated through whole irreversibility.
- $h^* = h + \frac{c^2}{2} + gZ$ is the so called “methalpy” – generalized enthalpy (Kestin, J., *A course in Thermodynamics*; Hemisphere: Washington, DC, USA, 1979; Volume 1, pp. 40 and 223)

2.2. Assumptions Considering Closed Thermal Systems

Let us suppose the general basic closed irreversible thermal systems, see Figure 2. The assumptions are

•no mass transfers

$$\sum_{\text{inlet}} \dot{m}h^* - \sum_{\text{outlet}} \dot{m}h^* = 0 \text{ and } -\sum_{\text{inlet}} \dot{m}s + \sum_{\text{outlet}} \dot{m}s = 0 \quad (9)$$

•non deformable boundary walls, and

$$-p_e \frac{\partial V}{\partial t} = 0 \quad (10)$$

•steady state operation

$$\frac{\partial U}{\partial t} = 0 \text{ and } \frac{\partial S}{\partial t} = 0 \quad (11)$$

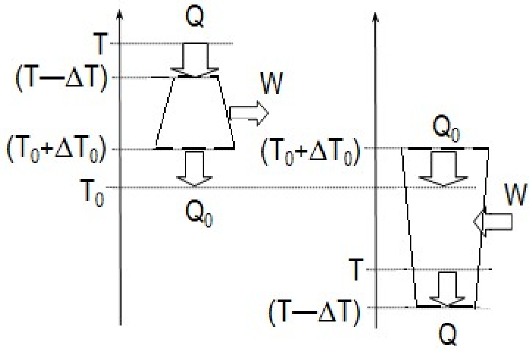

Figure 2. Scheme of the irreversible heat transfer interactions.

The associated entropy balance equation for the enlarged thermal system is, see Figure 2

$$-\mathrm{Irr}\frac{\dot{Q}}{T}+\frac{|\dot{Q}_0|}{T_0}=0 \tag{12}$$

The associated entropy balance equation only for the alone thermal system is, see Figure 2

$$-N_{irr}\frac{\dot{Q}}{T-\Delta T}+\frac{|\dot{Q}_0|}{T_0+\Delta T_0}=0 \tag{13}$$

The relation between Irr and N_{irr} is obtained from entropy balance Equations (12) and (13)

$$\mathrm{Irr}=N_{irr}\frac{T}{T_0}\frac{T_0+\Delta T_0}{T-\Delta T}=N_{irr}\theta_{HR}\theta_{mtt} \tag{14}$$

- $\dot{Q}, \dot{Q}_0$ are the heat transfer rates;
- $T, T_0, \Delta T, \Delta T_0$ are the mean temperatures of the heat source, of the heat sink and the corresponding mean log temperature differences controlling the heat transfers and $(T-\Delta T)$ and $(T_0+\Delta T_0)$ are the mean thermodynamic temperatures of the working fluid for the reversible heating and cooling processes;
- Irr is the comprehensive dimensionless irreversibility function linking the heat transfers through the entropy balance equation for the enlarged thermal system (both the external irreversibility and the internal one);
- N_{irr} is the internal dimensionless irreversibility function called as the number of internal irreversibility and linking the heat transfers through entropy balance equation only for the thermal system (only internal irreversibility);
- θ_{HR}, θ_{mtt} are the ratios of mean temperatures of external heat reservoirs and of mean thermodynamic temperatures of cycle’s non-adiabatic processes, i.e. reversible heating and cooling.

2.3. Irreversible Energy Efficiency of Enlarged Closed Thermal System

They will be demonstrated the comprehensive irreversible energy efficiency related to the complete reversible Carnot cycle, i.e., for the enlarged thermal system, see Figure 2.

• **Engine**

The delivered power of the enlarged engine cycle

$$\dot{W}_e = \dot{W}_e^{rev} + \dot{W}_{lost}^{irrev} = \dot{Q}_{rev}\left(1-\frac{T_0}{T}\right) - T_0\dot{S}_{gen}^{irrev} \tag{15}$$

The irreversible energy efficiency

$$\begin{aligned} EE_{engines}^{irrev} &= \frac{\dot{W}_e}{\dot{Q}_{rev}} = \frac{\dot{W}_e^{rev} + \dot{W}_{lost}^{irrev}}{\dot{Q}_{rev}} = \frac{\dot{W}_e^{rev}}{\dot{Q}_{rev}} + \frac{\dot{W}_{lost}^{irrev}}{\dot{Q}_{rev}} = 1 - \frac{T_0}{T} - \frac{T_0\dot{S}_{gen}^{irrev}}{\dot{m}(T-\Delta T)\Delta s_q} \\ &= 1-\frac{T_0}{T}\left(1+\frac{\theta_{SLT}\dot{S}_{gen}^{irrev}}{\dot{m}\Delta s_q}\right) = 1-\frac{T_0}{T}\mathrm{Irr} < EE_{Carnot} = 1-\frac{T_0}{T} \end{aligned} \tag{16}$$

where:

$\theta_{SLT} = \frac{T}{T-\Delta T}$ is a dimensionless temperature ratio related to the second law of thermodynamics; $\dot{m}\Delta s_q$ is the reversible entropy variation rate of the working fluid during the reversible heat input, $\dot{Q}_{rev}$, and $\left(1+\frac{\theta_{SLT}\dot{S}_{gen}^{irrev}}{\dot{m}\Delta s_q}\right) = \mathrm{Irr}$ is the primary form of the overall irreversibility dimensionless function.

- **Refrigeration unit**

The consumed power of the enlarged refrigeration unit

$$\left|\dot{W}_e\right| = -\dot{W}_e^{rev} - \dot{W}_{lost}^{irrev} = -\dot{Q}_{rev}\left(1 - \frac{T_0}{T}\right) + T_0\dot{S}_{gen}^{irrev} = \dot{Q}_{rev}\left(\frac{T_0}{T} - 1\right) + T_0\dot{S}_{gen}^{irrev} \tag{17}$$

The irreversible energy efficiency

$$EE_{refrigeration}^{irrev} = \frac{\dot{Q}_{rev}}{\left|\dot{W}_e\right|} = -\frac{\dot{Q}_{rev}}{\dot{W}_e^{rev} + \dot{W}_{lost}^{irrev}} = -\frac{1}{\frac{\dot{W}_e^{rev}}{\dot{Q}_{rev}} + \frac{\dot{W}_{lost}^{irrev}}{\dot{Q}_{rev}}} = \frac{1}{\frac{T_0}{T} - 1 + \frac{T_0\dot{S}_{gen}^{irrev}}{\dot{m}(T - \Delta T)\Delta s_q}}$$
$$= \frac{1}{\frac{T_0}{T}\left(1 + \frac{\theta_{SLT}\dot{S}_{gen}^{irrev}}{\dot{m}\Delta s_q}\right) - 1} = \frac{1}{\frac{T_0}{T}Irr - 1} < COP_{Carnot} = \frac{1}{\frac{T_0}{T} - 1} \tag{18}$$

where:

$\theta_{SLT} = \frac{T}{T-\Delta T}$ is a dimensionless temperature ratio related to the second law of thermodynamics; $\dot{m}\Delta s_q$ is the working fluid entropy variation rate during the reversible heat input $\dot{Q}_{rev}$, and $\left(1 + \frac{\theta_{SLT}\dot{S}_{gen}^{irrev}}{\dot{m}\Delta s_q}\right) = Irr$ is the primary form of the overall irreversibility dimensionless function.

2.4. Irreversible Energy Efficiency Only for the Closed Thermal System

They will be demonstrated the comprehensive irreversible energy efficiency related to the endoreversible Carnot cycle, see Figure 2.

- **Engine**

The delivered power of the alone closed engine cycle

$$\dot{W}_e = \dot{W}_e^{rev} + \dot{W}_{lost,cycle}^{irrev} = \dot{Q}_{rev}\left(1 - \frac{T_0 + \Delta T_0}{T - \Delta T}\right) - (T_0 + \Delta T_0)\dot{S}_{gen,cycle}^{irrev} \tag{19}$$

The irreversible energy efficiency

$$\begin{aligned} EE_{engines}^{irrev} &= \frac{\dot{W}_e}{\dot{Q}_{rev}} = \frac{\dot{W}_e^{rev} + \dot{W}_{lost,cycle}^{irrev}}{\dot{Q}_{rev}} = \frac{\dot{W}_e^{rev}}{\dot{Q}_{rev}} + \frac{\dot{W}_{lost,cycle}^{irrev}}{\dot{Q}_{rev}} \\ &= 1 - \frac{T_0 + \Delta T_0}{T - \Delta T} - \frac{(T_0 + \Delta T_0)\dot{S}_{gen,cycle}^{irrev}}{\dot{m}(T - \Delta T)\Delta s_q} = 1 - \frac{T_0 + \Delta T_0}{T - \Delta T}\left(1 + \frac{\dot{S}_{gen,cycle}^{irrev}}{\dot{m}\Delta s_q}\right) = 1 - \frac{T_0 + \Delta T_0}{T - \Delta T} N_{irr} \\ &< EE_{Carnot} = 1 - \frac{T_0 + \Delta T_0}{T - \Delta T} \end{aligned} \tag{20}$$

where:

$\dot{m}\Delta s_q$ is the working fluid entropy variation rate during the reversible heat input $\dot{Q}_{rev}$, and $\left(1 + \frac{\dot{S}_{gen,cycle}^{irrev}}{\dot{m}\Delta s_q}\right) = N_{irr}$ is the primary form of the internal irreversibility dimensionless function.

- **Refrigeration unit**

The consumed power of the alone closed refrigeration unit

$$\begin{aligned}|\dot{W}_e| &= -\dot{W}_e^{rev} - \dot{W}_{lost,cycle}^{irrev} = -\dot{Q}_{rev}\left(1 - \frac{T_0 + \Delta T_0}{T - \Delta T}\right) + (T_0 + \Delta T_0)\dot{S}_{gen,cycle}^{irrev} \\ &= \dot{Q}_{rev}\left(\frac{T_0 + \Delta T_0}{T - \Delta T} - 1\right) + (T_0 + \Delta T_0)\dot{S}_{gen,cycle}^{irrev}\end{aligned} \tag{21}$$

The irreversible energy efficiency

$$\begin{aligned}EE_{refrigeration}^{irrev} &= \frac{\dot{Q}_{rev}}{|\dot{W}_e|} = -\frac{\dot{Q}_{rev}}{\dot{W}_e^{rev} + \dot{W}_{lost,cycle}^{irrev}} = -\frac{1}{\frac{\dot{W}_e^{rev}}{\dot{Q}_{rev}} + \frac{\dot{W}_{lost,cycle}^{irrev}}{\dot{Q}_{rev}}} \\ &= \frac{1}{\frac{T_0 + \Delta T_0}{T - \Delta T} - 1 + \frac{(T_0 + \Delta T_0)\dot{S}_{gen,cycle}^{irrev}}{\dot{m}(T - \Delta T)\Delta s_q}} = \frac{1}{\frac{T_0 + \Delta T_0}{T - \Delta T}\left(1 + \frac{\dot{S}_{gen,cycle}^{irrev}}{\dot{m}\Delta s_q}\right) - 1} = \frac{1}{\frac{T_0 + \Delta T_0}{T - \Delta T}N_{irr} - 1} \\ &< COP_{Carnot} = \frac{1}{\frac{T_0 + \Delta T_0}{T - \Delta T} - 1}\end{aligned} \tag{22}$$

where:

$\dot{m}\Delta s_q$ is the working fluid entropy variation rate during the reversible heat input $\dot{Q}_{rev}$, and $\left(1 + \frac{\dot{S}_{gen,cycle}^{irrev}}{\dot{m}\Delta s_q}\right) = N_{irr}$ is the primary form of the internal irreversibility dimensionless function.

OBS1: The dimensionless functions, *Irr* and N_{irr}, depend on the reference entropy, $\Delta\dot{S} = \dot{m}\Delta s_q$ and on the corresponding $\dot{S}_{gen}^{irrev}$. At their turn, both parameters, $\Delta\dot{S} = \dot{m}\Delta s_q$ and $\dot{S}_{gen}^{irrev}$, will be strongly shaped through the working fluids nature and their thermodynamic properties.

OBS2. The heat rates exchanged with external heat reservoirs are the reversible heat rates for constant pressure processes where the irreversibility is defined by pressure drops, see Figs. 3 and 4.

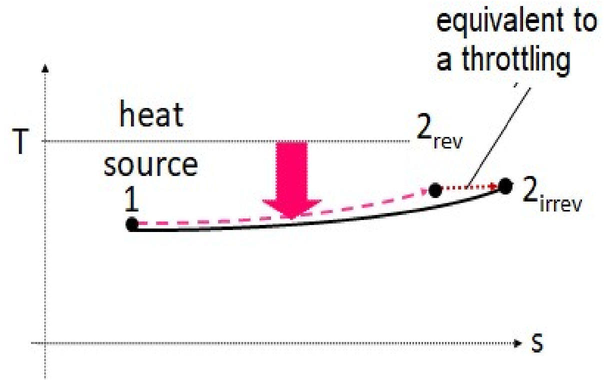

Fig. 3. The heat input for irreversible closed cycle

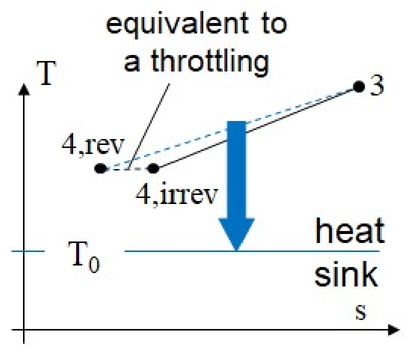

Fig. 4. The heat output for irreversible closed cycle

$$\dot{Q}_{12irrev} = \dot{m}(h_{2irev} - h_1) = \dot{m}(h_{2rev} - h_1) = \dot{m}T_{mq}^{12rev}(s_{2rev} - s_1)$$

$$\dot{Q}_{34irrev} = \dot{m}(h_{4irev} - h_3) = \dot{m}(h_{4rev} - h_3) = \dot{m}T_{mq}^{34rev}(s_{4rev} - s_3)$$

The reference entropy, $\Delta\dot{S} = \dot{m}\Delta s_q > 0$, is always the entropy variation of the working fluid during the cyclic reversible heating through the cyclic heat input. It must be mentioned that the reversible heat input is equalizing the irreversible one, because the extra irreversible entropy generation, caused by friction is corresponding to an equivalent throttling process. The same statement must be used for the cyclic heat output, see Figs 3 and 4.

When we have different irreversible non adiabatic processes, e.g. constant temperature, polytropic, constant volume we have to define the irreversibility either through adequate and known pressure drops caused by friction or through irreversible lost work alike it is defined the isentropic efficiency of an adiabatic process.

3. Design Imposed Operational Conditions

The analysis and design of irreversible cycles has two directions. The first one is to analyze the cycle ignoring the energy interactions with the environment by imposing either constant heat input or constant power or constant energy efficiency or constant reference entropy. The second one uses the energy interactions as main control functions and takes into consideration only the number of internal irreversibility as a general internal function quantifying the all internal irreversibility and linking the external heat transfers with external heat reservoirs.

3.1. FPDT Internal Design through Imposed Operational Conditions

They were evaluated the performances of a Joule-Brayton cycle working with two ideal gases, air and CO_2. The main finite physical dimension parameter was the classical compression ratio, π_C, and the dependence functions characterizing the performances of the irreversible cycle were:

- the maximum temperature on the cycle, T_{3irr} [K], see Figure 5,
- the energy efficiency, EE_{irr}, see Figure 6, and
- the number of internal irreversibility, N_{irr}, see Figure 7.

They were imposed the specific power $w = 500$ kJ/kg and the internal irreversible entropy generation known through isentropic efficiencies of compressor, η_{sC}, and of gas turbine, η_{sT}, and through the pressure drops inside exchangers, r_p. The all limitations would be controlled by the working fluids nature and by the magnitude of irreversibility.

3.1. Numerical results, internal design

Below are selected numerical results, see Figures 5, 6 and 7, for imposed constant power $w = 500$ kJ/kg and imposed irreversibility for graphs 1, 2, 3 and 4:

1: air, η_{sC} = 0.85, η_{sT} = 0.9, r_p = 0.975; **2**: air, η_{sC} = 0.8 and η_{sT} = 0.85, r_p = 0.95

3: CO_2, η_{sC} = 0.85, η_{sT} = 0.9, r_p = 0.975; **4**: CO_2, η_{sC} = 0.8 and η_{sT} = 0.85, r_p = 0.95

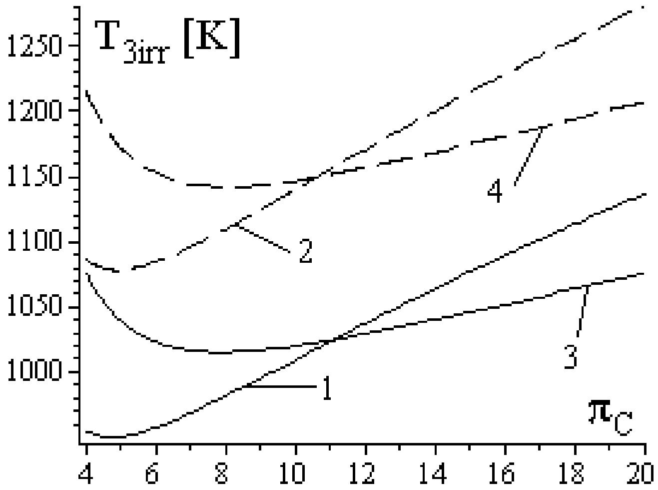

Figure 5. Dependences $T_{3irr} = f_T(\pi_C)$.

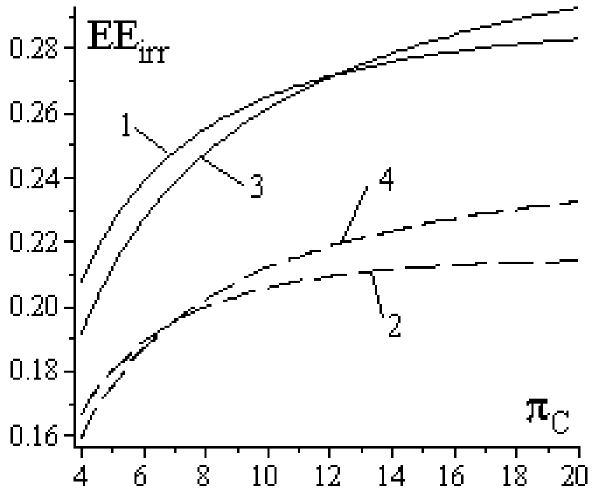

Figure 6. Dependences $EE_{irr} = f_E(\pi_C)$.

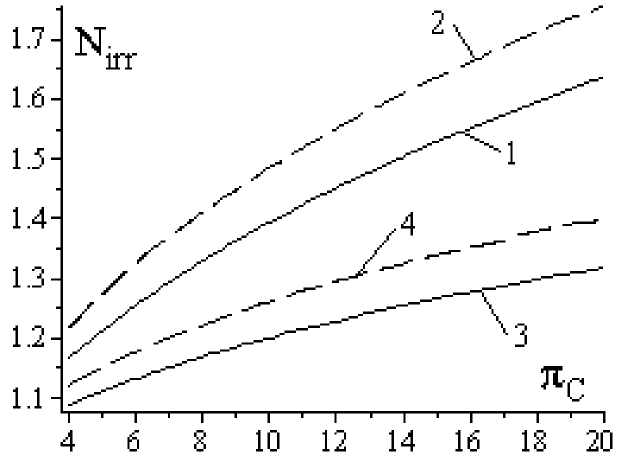

Figure 7. Dependences $N_{irr} = f_N(\pi_C)$.

4. Irreversible Trigeneration Cycles External Design Based on FPDT

This section is extending the mathematical models of external design to four irreversible closed trigeneration cycles:

a.engine cycle working in power mode and the reverse cycle working in refrigeration mode, the summer season;

b.engine cycle working in cogeneration mode and the reverse cycle working in refrigeration mode, the winter season;

c.engine cycle working in power mode and the reverse cycle working both in refrigeration mode and heat pump mode, the winter season; and

d. engine cycle working in cogeneration mode and the reverse cycle working both in refrigeration mode and heat pump mode, the winter season.

4.1. Basic Mathematical Model

The mathematical model joins the first law and the linear heat transfer law with the second law. The useful thermal energies must be known through the ratio of refrigeration rate to power (x) and the ratio of heating rate to power (y).

4.1.1. Engine Irreversible Cycle

The reference entropy variation rate is:

$$\Delta\dot{S}_E = \dot{m}\Delta s_q \tag{23}$$

The finite physical dimension control parameters are:

- Mean log temperature differences ΔTH [K] at the hot side and ΔTC [K] at the cold side.
- Thermal conductance $(UA)_H$ [kW/K] allocated to the hot side, and thermal conductance $(UA)_C$ [kW/K] allocated to the cold side.
- Thermal conductance inventory:

$$G_{TE} = G_H + G_C = (UA)_H + (UA)_C \ [kW \cdot K^{-1}] \tag{24}$$

$$g_H = \frac{G_H}{G_{TE}},\ g_C = \frac{G_C}{G_{TE}},\ g_H + g_C = 1,\ g_C = 1 - g_H \tag{25}$$

where U [$kW \cdot m^{-2} \cdot K^{-1}$] is the overall heat transfer coefficient and A [m^2] is the heat transfer area.

First Law Equations

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$$\dot{Q}_H = g_H G_{TE} \Delta T_H = T_H \Delta \dot{S}_E = (\theta_{HS} T_{CS} - \Delta T_H)\Delta \dot{S}_E \text{ at the hot side} \tag{26}$$

$$\overset{(26)}{\Rightarrow} \; G_{TE} = \frac{(\theta_{HS} T_{CS} - \Delta T_H)\Delta \dot{S}_E}{g_H \Delta T_H} \tag{27}$$

$$\dot{Q}_C = -(T_{CS} + \Delta T_C)\Delta \dot{S}_E N_{irr,E} = -(1 - g_H) G_{TE} \Delta T_C \text{ at the cold side} \tag{28}$$

$$\overset{(27,28)}{\Rightarrow} \quad \Delta T_C = \frac{g_H \Delta T_H N_{irr,E}}{\theta_{HS}\left\{1 - g_H - \frac{\Delta T_H\left[1 + g_H\left(N_{irr,E} - 1\right)\right]}{\theta_{HS} T_{CS}}\right\}} \tag{29}$$

$$\dot{W}_E = \dot{Q}_H + \dot{Q}_C = (\theta_{HS} T_{CS} - \Delta T_H)\Delta \dot{S}_E - \left(T_{CS} + \frac{g_H \Delta T_H N_{irr,E}}{\theta_{HS}\left\{1 - g_H - \frac{\Delta T_H\left[1 + g_H\left(N_{irr,E} - 1\right)\right]}{\theta_{HS} T_{CS}}\right\}}\right)\Delta \dot{S}_E N_{irr,E} \tag{30}$$

$$EE_{irr,E} = \frac{\dot{W}_E}{\dot{Q}_H} = 1 - \frac{\left(T_{CS} + \frac{g_H \Delta T_H N_{irr,E}}{\theta_{HS}\left\{1 - g_H - \frac{\Delta T_H\left[1 + g_H\left(N_{irr,E} - 1\right)\right]}{\theta_{HS} T_{CS}}\right\}}\right) N_{irr,E}}{(\theta_{HS} T_{CS} - \Delta T_H)} \tag{31}$$

The above performance functions get explicit forms if they are replacing the reference entropy through one imposed operational condition. The main difficulty is to correctly evaluate the possible imposed energy efficiency by a sensitivity analysis and to define the domain range of N_{irr}. The proof results are correlating the internal and external FPDT evaluations.

4.1.1. Numerical results, external design

As a very rapid computational example, they were imposed mixed operational conditions, constant power and constant energy efficiency: W = 100 kW, θ_{HS} = 4, T_{CS} = 323 K, and $EE_{irr,E}$ = 0.35, and some numbers of internal irreversibility, see Figures 8–10. The extra imposed energy efficiency allowed to find the relationship $\Delta TH=\phi(g_H,N_{irr,E})$:

$$\Delta T_H = 795.0769(1-g_H) \text{ with } N_{irr,E} = 1.00$$

$$\Delta T_H = \frac{670.846(1-g_H)}{1+0.25g_H} \text{ with } N_{irr,E} = 1.25$$

$$\Delta T_H = \frac{546.615(1-g_H)}{1+0.5g_H} \text{ with } N_{irr,E} = 1.50$$

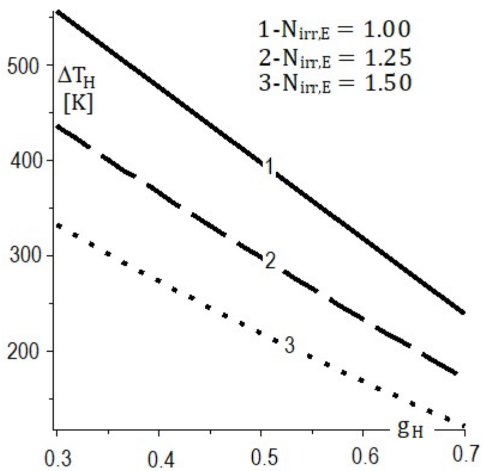

Figure 8. Dependence between the mean log temperature difference at the hot side and the dimensionless thermal conductance at the hot side, $\Delta T_H = f(g_H)$

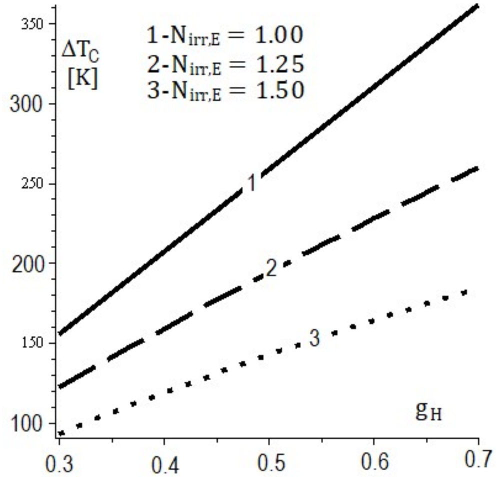

Figure 9. Dependence between the mean log temperature difference at the cold side and the dimensionless thermal conductance at the hot side $\Delta T_C = f(g_H)$

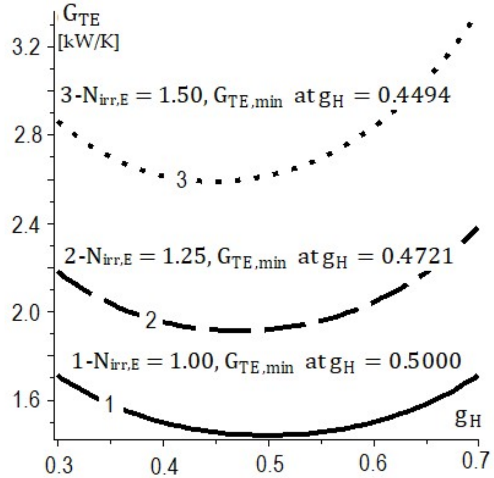

Figure 10. Dependence between the thermal conductance inventory and the dimensionless thermal conductance at the hot side $G_{TE} = f(g_H)$

4.1.2. Refrigeration Irreversible Cycle

•The reference entropy variation rate is:

$$\Delta\dot{S}_R = \dot{m}\Delta s_q \tag{32}$$

•The finite physical dimension control parameters are: mean log temperature differences ΔT_R [K] and ΔT_0 [K], inside of heat exchangers at the heat source and at the heat sink;

•Thermal conductances $(UA)_R$ inside the heat exchanger at the heat source, and $(UA)_0$ inside the heat exchanger at the heat sink:

•Thermal conductance inventory:

$$G_{TR} = G_R + G_0 = (UA)_R + (UA)_0 \text{ [kW·K}^{-1}\text{]} \tag{33}$$

$$g_R = \frac{G_R}{G_{TR}},\ g_0 = \frac{0}{G_{TR}},\ g_R + g_0 = 1,\ g_o = 1 - g_R \tag{34}$$

where U [$kW \cdot m^{-2} \cdot K^{-1}$] is the overall heat transfer coefficient and A [m^2] is the heat transfer area.

• First law balance equations:

$$\dot{Q}_R = g_R G_{TR} \Delta T_R = T_R \Delta \dot{S}_R = \left(\frac{T_{0S}}{\theta_{RS}} - \Delta T_R\right) \Delta \dot{S}_R \tag{35}$$

$$\overset{(34)}{\Rightarrow} \; G_{TR} = \frac{\left(\frac{T_{0S}}{\theta_{RS}} - \Delta T_R\right) \Delta \dot{S}_R}{G_{TR} \Delta T_R} \tag{36}$$

$$\dot{Q}_0 = -(1 - g_R) G_{TR} \Delta T_0 = -(T_{0S} + \Delta T_0) \Delta \dot{S}_R N_{irr,R} \tag{37}$$

$$\overset{(35,36)}{\Rightarrow} \; \Delta T_0 = \frac{g_R \theta_{RS} \Delta T_R N_{irr,R}}{1 - g_R - \frac{\theta_{RS} \Delta T_R}{T_{0S}} \left(1 + g_R\left(N_{irr,R} - 1\right)\right)} \tag{38}$$

$$\dot{W}_R = \dot{Q}_R + \dot{Q}_0 = \left[\left(\frac{T_{0S}}{\theta_{RS}} - \Delta T_R\right) - \left(T_{0S} + \frac{g_R \theta_{RS} \Delta T_R N_{irr,R}}{1 - g_R - \frac{\theta_{RS} \Delta T_R}{T_{0S}} \left(1 + g_R\left(N_{irr,R} - 1\right)\right)}\right) N_{irr,R}\right] \Delta \dot{S}_R \tag{39}$$

$$E\dot{E}_{irr,R} = \frac{\dot{Q}_R}{|\dot{W}_R|} = \frac{\frac{T_{0S}}{\theta_{RS}} - \Delta T_R}{\left(T_{0S} + \frac{g_R \theta_{RS} \Delta T_R N_{irr,R}}{1 - g_R - \frac{\theta_{RS} \Delta T_R}{T_{0S}} \left(1 + g_R\left(N_{irr,R} - 1\right)\right)}\right) N_{irr,R} - \left(\frac{T_{0S}}{\theta_{RS}} - \Delta T_R\right)} \tag{40}$$

The above performance functions get explicit forms if they are replacing the reference entropy through one imposed operational condition. The main difficulty is to correctly evaluate the possible imposed energy efficiency by a sensitivity analysis and to define the domain range of N_{irr}. The proof results are correlating the internal and external FPDT evaluations.

4.1.2. Numerical results, external design

As an example, they were imposed mixed operational conditions, constant heat input and constant energy efficiency: $\dot{Q}_R = 0.1\dot{W}_E = 10$ kW, T_{RS} = 263 K, T_{0S} = 323 K, $EE_{irr,R}$ = COP = 2, see Figures 11–13. The extra imposed energy efficiency allowed to find the first explicit operational function $\Delta T_R = f(g_R, N_{irr,R})$.

$$\Delta T_R = \frac{143(1-g_R)}{3} \text{ with } N_{irr,E} = 1.00$$

$$\Delta T_R = \frac{78.4(1-g_R)}{3+0.3g_R} \text{ with } N_{irr,E} = 1.10$$

$$\Delta T_R = \frac{13.8(1-g_R)}{3+0.6g_R} \text{ with } N_{irr,E} = 1.20$$

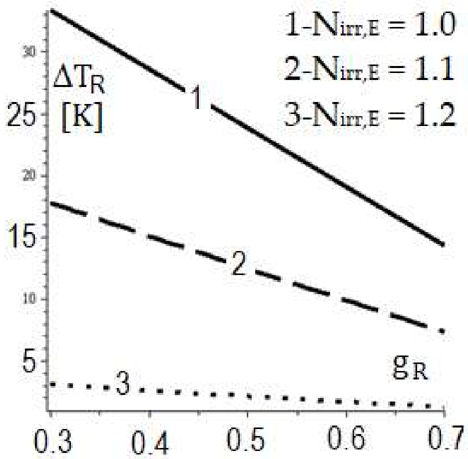

Figure 11. Dependence between the mean log temperature difference at the cold side and the dimensionless thermal conductance at the cold side, $\Delta T_R = f(g_R)$

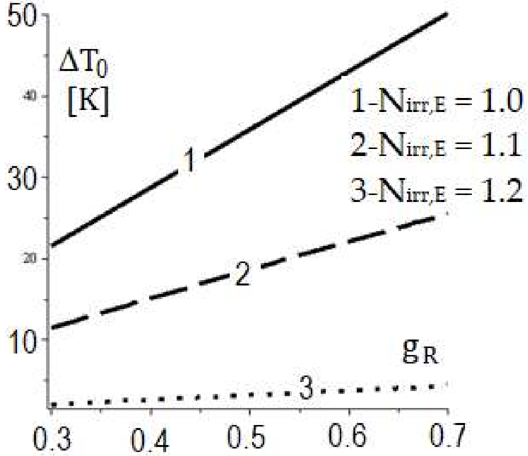

Figure 12. Dependence between the mean log temperature difference at the hot side and the dimensionless thermal conductance at the cold side, $\Delta T_0 = f(g_R)$.

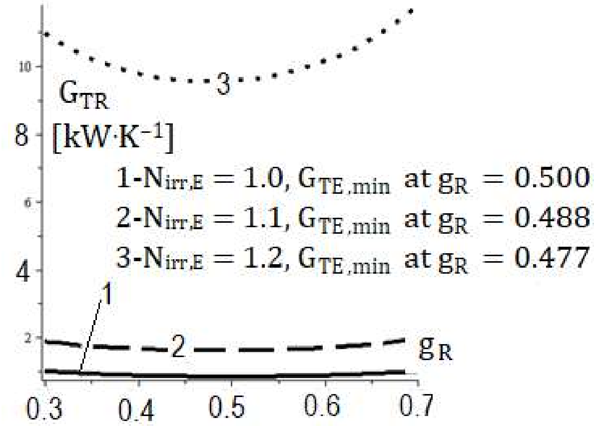

Figure 13. Dependence between the thermal conductance inventory and the dimensionless thermal conductance at the cold side ide, $G_{TR} = f(g_R)$

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4.2. Energy Efficiency of Irreversible Trigeneration System

- Case "a"—energy efficiency:

$$EE_a = \frac{\dot{W}_E - |\dot{W}_R| + \dot{Q}_R}{\dot{Q}_H} = EE_E\left(1 + x\frac{COP - 1}{COP}\right) \quad (41)$$

- Case "b"—energy efficiency:

$$EE_b = \frac{\dot{W}_E - |\dot{W}_R| + \dot{Q}_R + |\dot{Q}_C|^*}{\dot{Q}_H} = EE_{cog} + EE_E x\frac{COP - 1}{COP} \quad (42)$$

- Case "c"—energy efficiency:

$$EE_c = \frac{\dot{W}_E - |\dot{W}_R| + \dot{Q}_R + |\dot{Q}_0|}{\dot{Q}_H} = EE_E(1 + 2x) \quad (43)$$

- Case "d"—energy efficiency:

$$EE_d = \frac{\dot{W}_E - |\dot{W}_R| + \dot{Q}_R + |\dot{Q}_C|^* + |\dot{Q}_0|}{\dot{Q}_H} = EE_{cog} + 2EE_E x \quad (44)$$

They must emphasize that Equations (41)–(44) are identical for ideal reversible, endoreversible and irreversible trigeneration systems. They have to know the real energy efficiencies of system components, i.e., EE_{cog}, $EE_{E,real}$, and COP_{real}—and ratio x.

For all cases, the minimum useful power compels the maximum *x* ratio

$$\dot{W}_u = \dot{W}_E - |\dot{W}_R| = \dot{W}_E\left(1 - \frac{x}{COP}\right) \geq \dot{W}_{u,min} \Rightarrow x \leq COP\left(1 - \frac{\dot{W}_{u,min}}{\dot{W}_E}\right) \quad (45)$$

Table 1. External parameters for the engine, $\dot{W}_E = 100$ kW, EE_E = 0.35 (imposed), θ_{HS} = 4 (imposed)

Trigeneration	N_{irr}	g_H	T_{CS} (K)	ΔT_H (K)	ΔT_C (K)	G_{TE} ($kW \cdot K^{-1}$)
(a)	1.00	0.5000	308	379	246	1.507
	1.25	0.4721	308	302	176	2.003
	1.50	0.4494	308	234	124	2.713
(b)	1.00	0.5000	343	422	274	1.354
	1.25	0.4721	343	336	196	1.799
	1.50	0.4494	343	261	138	2.436
(c)	1.00	0.5000	273	336	218	1.701
	1.25	0.4721	273	268	156	2.261
	1.50	0.4494	273	208	110	3.061
(d)	1.00	0.5000	343	422	274	1.354
	1.25	0.4721	343	336	196	1.799
	1.50	0.4494	343	261	138	2.436

Table 2. External parameters for the refrigeration unit, $\dot{Q}_R = 10\ \mathrm{kW}$

Trigeneration	N_{irr}	g_R	T_{0S} (K)	T_{RS} (K)	ΔT_R (K)	ΔT_0 (K)	G_{TR} ($kW{\cdot}K^{-1}$)	COP
(a)	1.0	0.500	308	253	23.83	35.75	0.839	2
	1.1	0.488	308	253	13.24	18.83	1.547	2
	1.2	0.477	308	253	3.15	4.31	6.503	2
(b)	1.0	0.500	273	253	24.13	32.17	0.829	3
	1.1	0.488	273	253	13.56	17.13	1.511	3
	1.2	0.477	273	253	3.49	4.24	5.88	3
(c)	1.0	0.500	343	253	23.60	39.33	0.875	1.5
	1.1	0.488	343	253	13.00	20.53	1.577	1.5
	1.2	0.477	343	253	2.88	4.38	7.106	1.5
(d)	1.0	0.500	343	253	23.60	39.33	0.875	1.5
	1.1	0.488	343	253	13.00	20.53	1.577	1.5
	1.2	0.477	343	253	2.88	4.38	7.106	1.5

The comparison of different kind of trigeneration systems might be assessed only if they have similar operational features, see for instance Figure 14.1 with: $EE_{irr,E} = 0.35$, and $COP = 2$, and $EE_{cog} = 0.85$ and the minimum useful power 50% from engine power, i.e., $x_{max} = 1$.

In Figure 14.2 are compared the ideal reversible energy efficiency for ideal trigeneration cycle built with ideal Carnot cycles, with $EE_E = 0.75$ for $\theta_{HS} = 4$ as in Table 1, and $COP = T_{RS}/(T_{0S} - T_{RS})$ with temperatures from Table 2, and $EE_{cog} = 1$ and the minimum useful power 50% from engine power, i.e., $x_{max} = 1$.

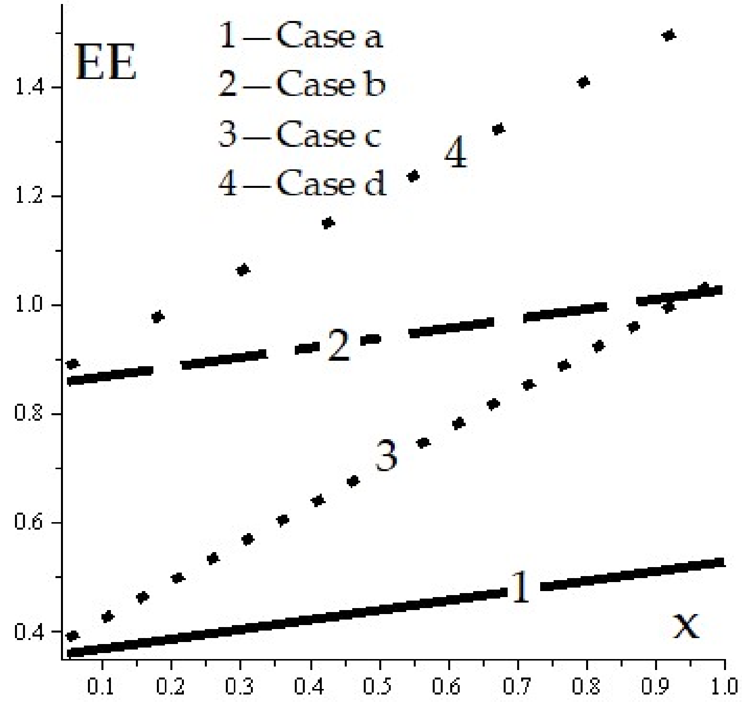

Figure 14.1. The irreversible energy efficiency of trigeneration systems

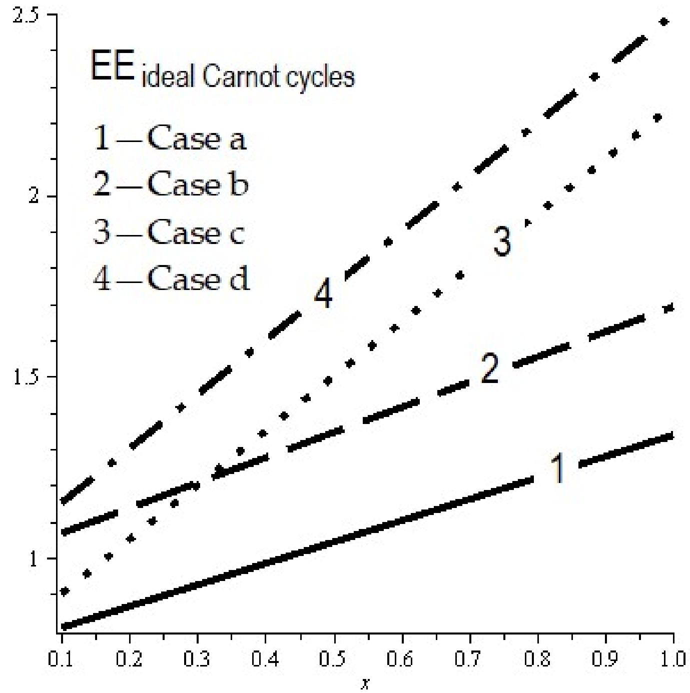

Figure 14.2. The ideal reversible energy efficiency of trigeneration systems

5. Conclusions

The generalizing FPDT mathematical models minimizes the finite physical dimensions external control parameters, and operational corresponding dependence functions of engine and refrigeration cycles included in a trigeneration system.

There are two kind of control parameters, four external and two internal. The four external control parameters are pertaining to external heat transfer—i.e., two mean log temperature differences and two dimensionless thermal conductance inventories. The internal ones are the reference entropy and the number of internal irreversibility which delineate a single dimensionless concept a priori evaluating the accumulated internal irreversibility.

The reference entropy function is replaced through the operational adopted condition—i.e., either through the imposed power, or through the imposed heat input as in this paper, or through the imposed energy efficiency or through the imposed reference entropy.

The number of internal irreversibility is a dimensionless parameter generalizing the evaluation of accumulated irreversible entropy generated along the cycle.

Before each FPDT work they must be defined the operational possible domain range of the number of internal irreversibility depending on the working fluid nature and on the thermal system type.

The evaluated specific numerical results showed as higher the internal irreversibility as lower the external irreversibility in order to maintain constant energy efficiency.

The Equations (41) to (44) are universal, can be applied for ideal reversible trigeneration cycle, endoreversible, or irreversible ones, see for instance Figures 14.1, 14.2. The comparison reversible–endoreversible–irreversible has to use the operational similarity and thus they can be completed various analyses and optimizing assessments.

THANK YOU!